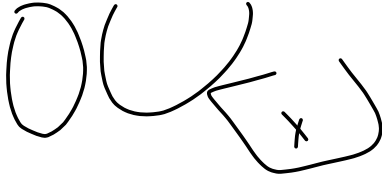


Unit 3



Lesson 1/ Similarity of Polygons

prep 3

First term 2027

طريقك للأمان ... الأيسر في الماث

Mr. Abanoub



01097997401



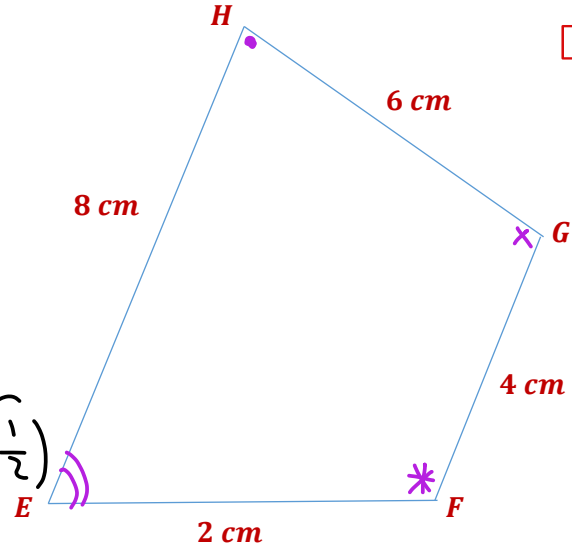
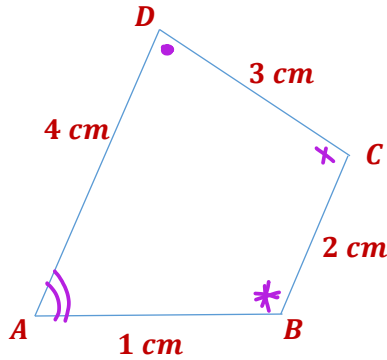
Similarity of two polygons

Two polygons M_1 & M_2 are said to be similar if the following two conditions satisfied together.

- ✓ Their corresponding angles are equal in measure.
- ✓ The length of their corresponding sides is proportional.

In this case, the polygon $M_1 \sim$ the polygon M_2





$$\frac{AB}{EF} = \frac{1}{2}$$

$$\frac{BC}{FG} = \frac{2}{4} = \left(\frac{1}{2}\right)$$

$$\frac{CD}{GH} = \frac{3}{6} = \left(\frac{1}{2}\right)$$

$$\frac{AD}{EH} = \frac{4}{8} = \left(\frac{1}{2}\right)$$

$$\therefore \frac{AB}{EF} = \frac{BC}{FG} = \frac{CD}{GH} = \frac{AD}{EH}$$

similarity ratio $= \left(\frac{1}{2}\right) < 1$

$$m(\angle A) = m(\angle E)$$

$$m(\angle B) = m(\angle F)$$

$$m(\angle C) = m(\angle G)$$

$$m(\angle D) = m(\angle H)$$

(11)

$$\frac{EF}{AB} = \frac{2}{1} = 2$$

$$\frac{FG}{BC} = \frac{4}{2} = 2$$

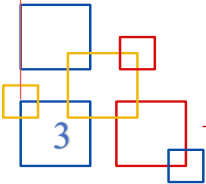
$$\frac{GH}{CD} = \frac{6}{3} = 2$$

$$\frac{EH}{AD} = \frac{8}{4} = 2$$

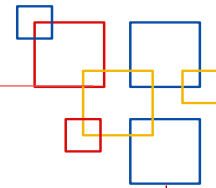
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AD

similarity ratio $\Rightarrow 2 > 1$ 

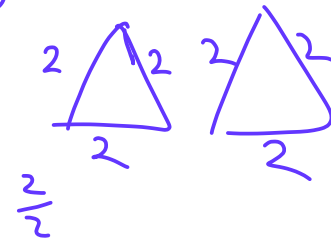
Senior 1

If $ABCD \sim EFGH$

Similarity Ratio (k)

It is a ratio between the length of two corresponding sides in similar polygons.

- 1 If $k > 1$, the polygon M_1 is an **enlargement** of polygon M_2 .
- 2 If $k < 1$, the polygon M_1 is a **Shrinking** of polygon M_2 .
- 3 If $k = 1$, the polygon M_1 is a **Congruent** of polygon M_2 .



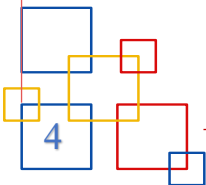
Note

If polygon $ABCD \sim$ polygon $EFGH$ then

$$\frac{AB}{EF} = \frac{BC}{FG} = \frac{CD}{GH} = \frac{DA}{HE} = k$$

Where k is the similarity ratio of polygon $ABCD$ to polygon $EFGH$

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For example

- If polygon ABCD ~ polygon EFGH and

$$\frac{AB}{EF} = \frac{BC}{FG} = \frac{CD}{GH} = \frac{DA}{HE} = \frac{3}{2}$$

- Then the similarity ratio of polygon ABCD to polygon EFGH is equal to $\frac{3}{2}$
- The similarity ratio of polygon EFGH to polygon ABCD is equal to $\frac{2}{3}$

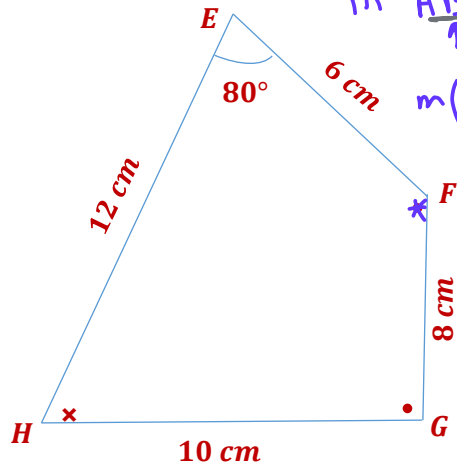
Ex.1

In each of the following, does polygon **ABCD** is similar to polygon **EFGH** or not?

State the ratio of similarity of polygon **ABCD** to polygon **EFGH** if they are similar.

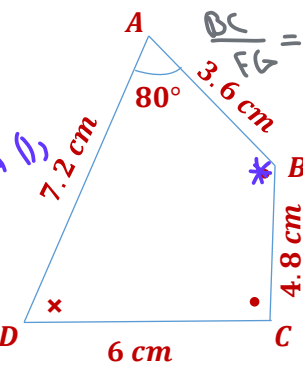
$$\triangle FGH \sim \triangle ABCD \Rightarrow \frac{5}{3}$$

1



In $\triangle ABCD$, $\triangle FGH$

$$\begin{aligned} m(\angle A) &= m(\angle E) \\ m(\angle B) &= m(\angle F) \\ m(\angle C) &= m(\angle G) \\ m(\angle D) &= m(\angle H) \end{aligned}$$



$$\frac{AB}{EF} = \frac{3.6}{6} = \left(\frac{3}{5}\right)$$

$$\frac{BC}{FG} = \frac{4.8}{8} = \left(\frac{3}{5}\right)$$

$$\frac{CD}{GH} = \frac{6}{10} = \left(\frac{3}{5}\right)$$

$$\frac{AD}{EH} = \frac{7.2}{12} = \left(\frac{3}{5}\right)$$

from (1), (2) $\therefore$ polygon **ABCD** $\sim$ polygon **EFGH**

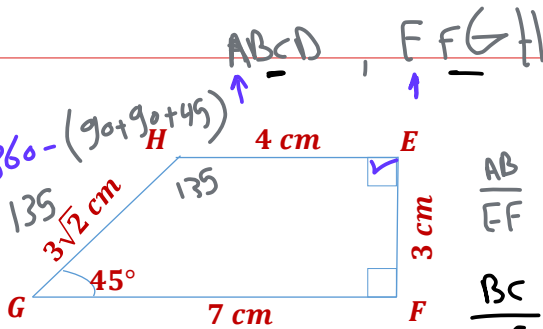
$$\therefore \frac{AB}{EF} = \frac{BC}{FG} = \frac{CD}{GH} = \frac{AD}{EH}$$

$$01097997401 = \frac{3}{5}$$



2

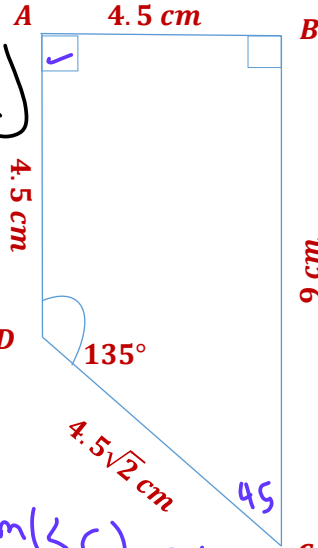
$$m(\angle A) = 360 - (90 + 90 + 45) = 135$$



$$\frac{AB}{EF} = \frac{4.5}{3} = \left(\frac{3}{2}\right)$$

$$\frac{BC}{FG} = \left(\frac{9}{7}\right)$$

$$\therefore \frac{AB}{EF} \neq \frac{BC}{FG}$$



In polygons ABCD, FFGH

$$m(\angle A) = m(\angle F)$$

$$m(\angle B) = m(\angle E)$$

$$m(\angle C) = m(\angle G)$$

$$m(\angle D) = m(\angle H)$$

(1)

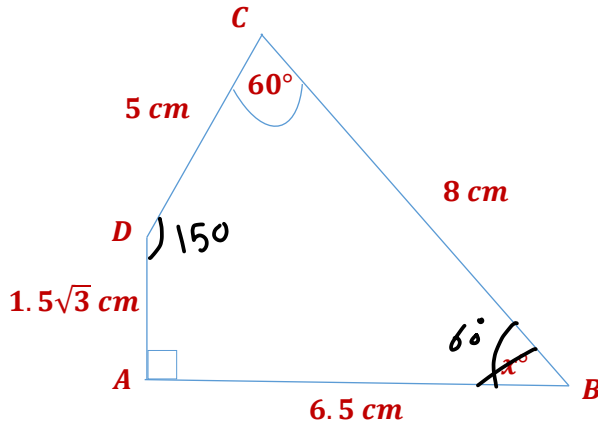
The length of $m(\angle C) = 360 - (90 + 90 + 135) = 45$
 Corresponding sides are not proportional.

$\therefore$ The measures of corresponding angles are equal

$\therefore$ polygon ABCD is not similar to polygon EFGH

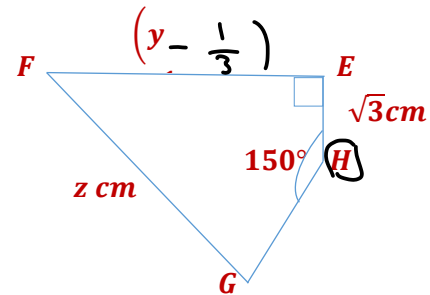
Ex:2

In each the opposite figure, polygon **ABCD** is similar to polygon **EFGH**
Find (x, y, z)



From similarity. $\therefore m(\angle D) = m(\angle H)$
 $= 150^\circ$

$$\therefore x = 360 - (60 + 150 + 90) = 60^\circ$$



$$\therefore \frac{AB}{EF} = \frac{BC}{FG} = \frac{CD}{GH} = \frac{AD}{EH}$$

$$\frac{6.5}{y - \frac{1}{3}} = \frac{8}{z} = \frac{5}{\cancel{GH}} = \frac{1.5\sqrt{3}}{\sqrt{3}} \cdot \frac{3}{2}$$

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$$\frac{6.5}{y - \frac{1}{3}} = \frac{3}{2}$$

$$y - \frac{1}{3} = \frac{6.5 \times 2}{3}$$

$$y - \frac{1}{3} = \frac{13}{3} \quad (\text{shift 5-d})$$

$$y - \frac{1}{3} = 4\frac{1}{3}$$

$$\therefore y = 4\frac{1}{3} + \frac{1}{3}$$

$$y = 4\frac{2}{3}$$

$$\frac{8}{2} = \frac{3}{2}$$

$$2 = \frac{8 \times 2}{3}$$

$$2 = \frac{16}{3}$$

$$2 = 5\frac{1}{3}$$

$$\frac{1.5 \sqrt{3}}{\sqrt{3}} = \left(\frac{3}{2}\right)$$

Remark

The ratio between the perimeters of two similar polygons = the ratio between the length of two corresponding sides of them. *ملاحظة*

similar polygons

ملاحظة*

$$\frac{\text{the perimeter of the polygon}_1}{\text{the perimeter of the polygon}_2} = \frac{\text{the length of the polygon}_1 \times \text{side}}{\text{the length of the polygon}_2 \times \text{side}}$$



$$\frac{3}{6} = \left(\frac{1}{2}\right)$$

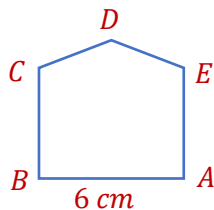
$$\frac{1+3+4}{2+6+8} = \frac{8}{16} = \left(\frac{1}{2}\right)$$

Try

In the opposite figure:

The polygon **ABCDE** is similar to the polygon **KFGHJ**

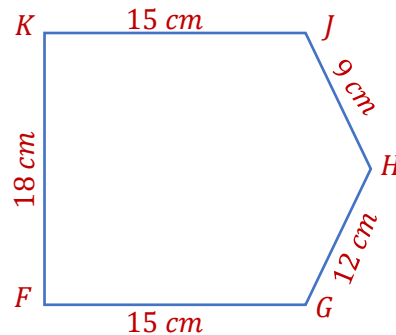
Calculate the perimeter of the polygon **ABCDE**



The perimeter of polygon ABCDE

$$\frac{\text{The perimeter of polygon KFGHJ}}{69} = \frac{6}{18}$$

$$\therefore \text{The perimeter of polygon ABCDE} = \frac{69 \times 6}{18} = 23 \text{ cm}$$



$$\begin{aligned} \text{Perimeter of polygon KFGHJ} &= 15 \times 2 + 12 + 18 + 9 \\ &= 30 + 30 + 9 \\ &= 69 \text{ cm} \end{aligned}$$

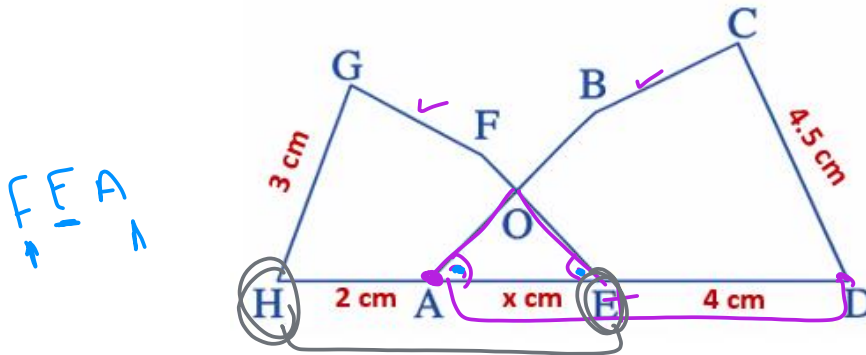
Ex.3

In the opposite figure:

Polygon **ABCD** ~ polygon **EFGH**

- (1) Prove that: $OA = OE$
- (2) Find the value of x
- (3) If the perimeter **ABCD** = **19 cm**

Find the perimeter of polygon **EFGH**



From similarity $\Rightarrow m(\angle BAE) = m(\angle FEA)$

$\therefore \triangle OAF$ is an isosceles triangle

$\therefore OA = OF$

(first req)

$\overline{ABCD} \sim \overline{EFGH}$

$$\frac{AB}{EF} = \frac{BC}{FG} = \frac{CD}{GH} = \frac{AD}{FH} \Rightarrow$$

$$\frac{4.5}{3} = \frac{X+4}{X+2}$$

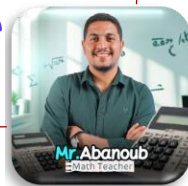
From similarity

$$\frac{\text{Perimeter of } ABCD}{\text{Perimeter } EFGH} = \frac{CD}{GH}$$

$$\frac{19}{X} = \frac{3}{2}$$

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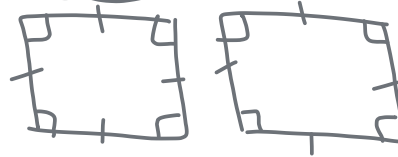
$$\begin{aligned} \frac{3}{2} &= \frac{X+4}{X+2} \\ 3X+6 &= 2X+8 \\ 3X-2X &= 8-6 \\ X &= 2 \end{aligned}$$



$$x = \frac{19 \times 2}{3} = \frac{38}{3} = 12 \frac{2}{3} \text{ cm}$$

Note

- Congruent polygons are similar, but similar polygons are not necessarily congruent.

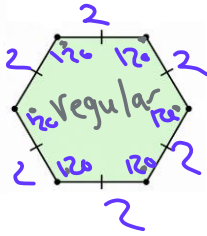
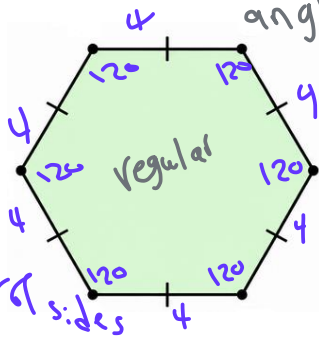


- Regular polygons that have the same number of sides are always similar.

So

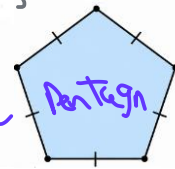
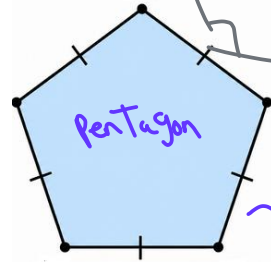
✓ Sides are equal

angles are equal in measure

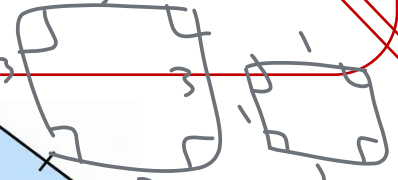


number of sides $\uparrow$

$$\frac{n-2}{n} \times 180$$



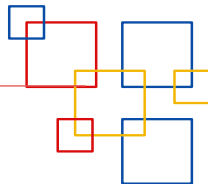
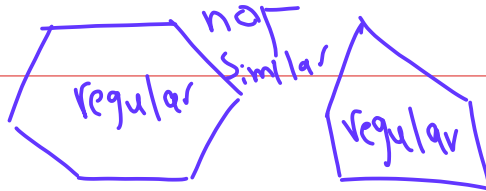
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Ex. 4



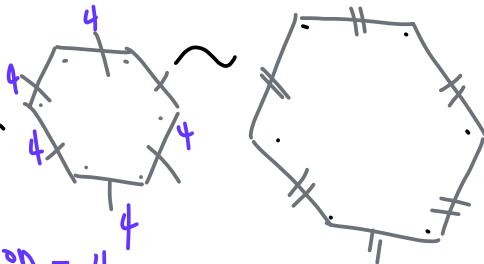
Two regular hexagons have a similarity ratio of the first to the second of $\frac{1}{3}$ and the side length of the first is 4 cm, what is the perimeter of the second?

∴ Two hexagons are regular

∴ Two hexagons are similar

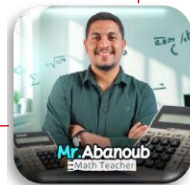
∴ side length of the first hexagon = 4 cm

∴ perimeter of the first hexagon = $4 \times 6 = 24$ cm



~~perimeter of the first hexagon = 24~~
~~perimeter of the second hexagon = 72~~
 $\frac{1}{3}$

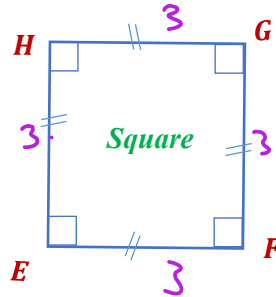
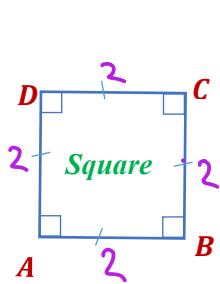
perimeter of the second regular hexagon = $3 \times 24 = 72$ cm



Similarity of Special Quadrilaterals

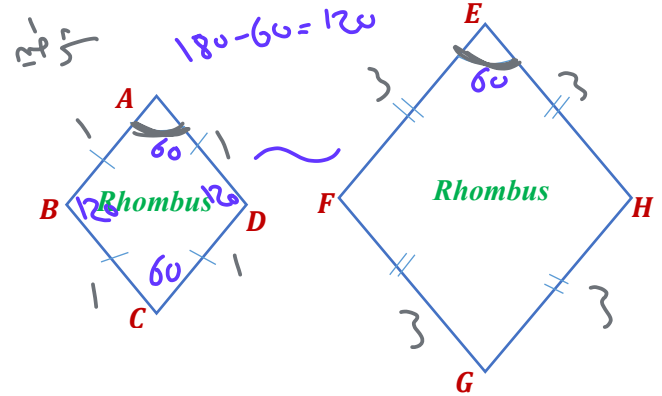
All squares are similar.

الجميع متشابه



$EFGH \sim ABCD$

Two rhombuses are similar if the measures of two corresponding angles in them are equal.

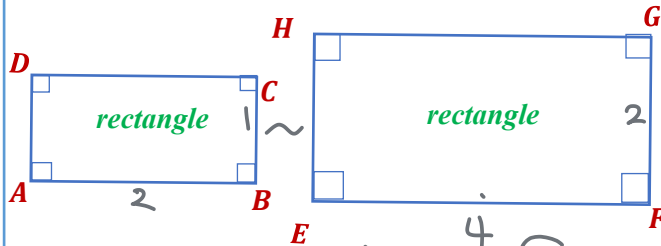


If: $m(\angle A) = m(\angle E)$
So: $EFGH \sim ABCD$

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Two rectangles are similar if their corresponding dimensions are proportional.

length, width



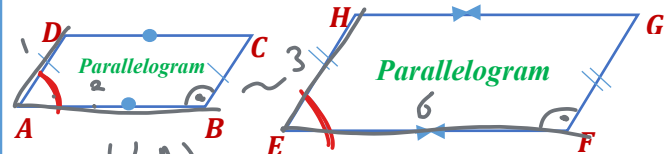
If: $\frac{AB}{EF} = \frac{BC}{FG}$

So: $EFGH \sim ABCD$

$$\frac{AB}{EF} = \frac{2}{4} = \left(\frac{1}{2}\right)$$

$$\frac{BC}{FG} = \left(\frac{1}{2}\right)$$

Two parallelograms are similar if the measure of an angle in one equals the measure of an angle in the other and the lengths of the sides containing these two angles are proportional.



$$m(\angle A) = m(\angle E)$$

If: $\frac{AB}{EF} = \frac{BC}{FG}$

So: $EFGH \sim ABCD$

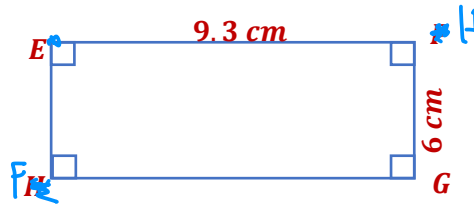
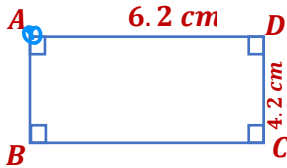
$$\frac{AB}{EF} = \frac{2}{6} = \left(\frac{1}{3}\right)$$

$$\frac{AD}{EH} = \left(\frac{1}{3}\right)$$

Ex. 5

In each of the following, determine whether two polygons are similar or not-similar, and find the similarity ratio of polygon **ABCD** to polygon **EFGH** if they are similar.

① Rectangles: **ABCD**, **EFGH** length . width



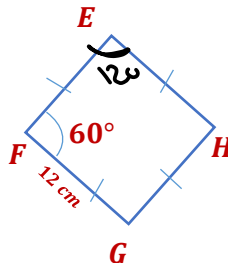
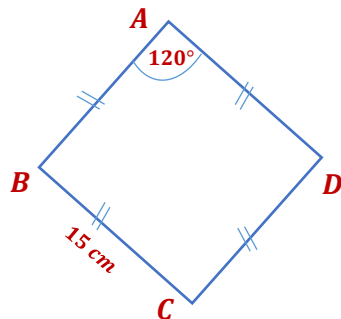
$$\frac{AD}{EH} = \frac{6.2}{9.3} = \left(\frac{2}{3}\right)$$

$$\therefore \frac{AD}{EH} \neq \frac{CD}{GH}$$

$$\frac{CD}{GH} = \frac{4.2}{6} = \left(\frac{7}{10}\right)$$

$\therefore$ rectangle ABCD is not similar to rectangle EFGH.

② Rhombuses: $ABCD, EFGH$



Each two consecutive angle are supplementary

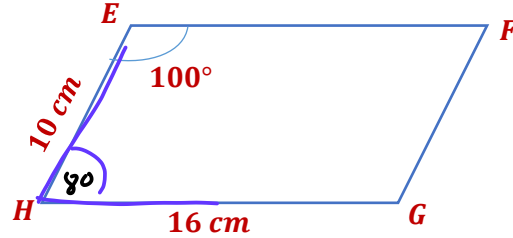
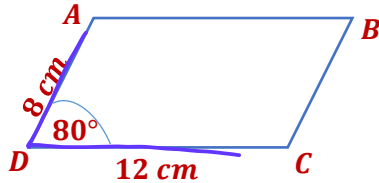
In Rhombus $EFGH$

$$\therefore m(\angle F) = 180 - 60 = 120^\circ$$

$$\therefore m(\angle A) = m(\angle F)$$

$\therefore$ Rhombus $ABCD \sim$ Rhombus $EFGH$

③ Parallelograms: $ABCD$, $EFGH$



In parallelograms
 $ABCD$, $EFGH$

In Rhombus $EFGH$

$$\therefore m(\angle H) = 180 - 100 = 80^\circ$$

$$m(\angle D) = m(\angle H) = 80$$

$$\frac{DC}{CH} = \frac{12}{16} = \left(\frac{3}{4}\right)$$

$$\frac{EH}{FH} = \frac{8}{10} = \left(\frac{4}{5}\right)$$

$$\therefore \frac{DC}{CH} \neq \frac{EH}{FH}$$

$\therefore$ Parallelogram $ABCD$ is not similar to $EFGH$.